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Non-BPS brane cosmology

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Abstract: We study cosmology on a BPS D3-brane evolving in the 10D SUGRA background describing a non-BPS brane. Initially the BPS brane is taken to be a probe whose dynamics we determine in the non-compact non-BPS background. The cosmology observed on the brane is of the FRW type with a scale factor $S(\tau)$. In this mirage cosmology approach, there is no self-gravity on the brane which cannot inflate. Self-gravity is then included by compactifying the background space-time. The low energy effective theory below the compactification scale is shown to be bi-metric, with matter coupling to the geometrically induced metric on the brane. The geometrical scale factor on the brane is now $S(\tau)a(\tau)$ where $a(\tau)$ arises from brane self-gravity. In this non-BPS scenario the brane generically inflates. We study the resulting inflationary scenario taking into account the fact that the non-BPS brane eventually decays on a time-scale much larger than the typical inflationary time-scale. After the decay, the theory ceases to be bi-metric and COBE normalization is used to estimate the string scale which is found to be of order 10^{14} GeV.

Keywords: D-branes, Tachyon Condensation, Cosmology of Theories beyond the SM, Physics of the Early Universe.

Contents

1. Introduction

Since the advent of branes in string theory, brane models have been used to improve our understanding of physics beyond the standard model. Brane constructions have proved to be particularly successful in providing valuable insights for such arduous puzzles as the cosmological constant problem and the hierarchy problem. In their most common guise, models of this type are five-dimensional following the work of Randall-Sundrum [1, 2] and the compactification of the heterotic M-Theory on a Calabi-Yau manifold [3]. Our universe is then one of the boundaries of the fifth dimension.

Another approach has been followed starting from ten-dimensional string theory compactified on a large volume manifold. These models allow one to lower the string scales in such a way that stringy effects may become observable at accelerators. As opposed to the five-dimensional approach the geometry of space-time is flat [4, 5].

One of the puzzles of high energy physics is supersymmetry breaking. A particularly useful way of breaking supersymmetry has been studied with branes, i.e. configurations of branes and anti-branes do not preserve any supersymmetry because of the incompatibility of their respective BPS conditions. The lack of supersymmetry implies the existence of a tachyon in the open string spectrum, signaling an instability [6]–[16]. Indeed brane-antibrane configurations are not static due the existence of a long-range force between them. Such a force can be evaluated in the large separation regime, and hence at low energy the brane separation can be viewed as a field with a non-trivial potential. This setting has been used to model inflation in the primordial universe [17]–[32]. As long as the brane and anti-brane are far apart one can trust the supergravity approximation. When the branes are nearly colliding, the open string description is more appropriate. In that case, the Sen conjecture [9]ascertains that the open string tachyon condensates leaving behind a supersymmetric configuration. For brane-antibrane, the result of the annihilation can either be flat empty space or higher codimension supersymmetric branes [12]. This idea has been used to justify the end of inflation and the occurrence of reheating.

One may also be interested in non-BPS branes per se, i.e. branes which breaks all supersymmetries. These objects can be constructed in string theory at small string coupling by considering a brane-antibrane configuration after modding out $(-1)^{F_L}$, the left fermion number [11]. This gives rise to non-BPS branes with a real tachyon in the open string spectrum. As brane-antibranes, they are meant to decay via open string tachyon condensation. Nevertheless one may be interested in describing the non-BPS branes from the supergravity point of view, i.e. the classical configuration of the closed string zero modes which results from the distortion of space-time by the presence of a non-BPS brane. Such a programme has been carried out in [33]. The tachyon condensation becomes a path in the parameter space interpolating between the non-BPS branes and the BPS branes.

In this paper we consider the low energy physics viewed on a probe brane in the background of a non-BPS brane. The probe is taken to be far enough away from the non-BPS brane that the supergravity approximation can be trusted, and analyse the dynamics before the decay of the non-BPS brane. We consider two situations. In section 2, we study the case of a D3-brane probing the geometry of a non-BPS brane in non-compact ten-dimensional space. In that case there is no self-gravity on the brane, though there is an effective cosmology. We show that despite the attraction between the branes, inflation never occurs. Then in section 3 we compactify the model on a large volume six-manifold that we do not need to specify. We will consider that the moduli resulting from the compactification have been stabilized by an unknowm mechanism. In particular the overall volume of compactification will taken to be a free parameter of the model. As a result of self-gravity, the effective four-dimensional Planck mass is finite and the brane undergoes a phase of inflation. After the decay of the non-BPS brane, the probe brane reheats. Inflation does not require any fine-tuning of the parameters of the non-BPS brane, i.e. it takes place whether the non-BPS brane is at the beginning or end of its decay. We stress that the only fine-tuning comes from the initial position of the probe which determines the number of e-folds of inflation.

In section 2, where the geometry is non-compact, the brane "mirage" cosmology arises solely due to the brane motion [35]–[38]. The result of compactification, in section 3, leads us to consider a 4d effective action similar to that of [17]–[32] which includes self-gravity. However, as opposed to those references, our theory on the brane turns out to be bi-metric. The setup we envisage is also different to that of [20, 27] who suppose that inflation ends in a $\bar{\mathrm{D}}3-\mathrm{D}3$ collision which will generically produce branes of higher codimension. Here the non-BPS brane eventually decays leaving only our D3-brane and radiation. The scenario again differs from that of [21, 30] who considered non-parallel branes, and of reference [22] in which inflation occurs during a $\bar{\mathrm{D}}5-\mathrm{D}5$ annihilation to produce three-branes.

2. Non-BPS background metric

In the supergravity approximation, where only massless closed string modes are taken into account, the background generated by a non-BPS p-brane at the origin was constructed in [33, 34]. This background metric, in the Einstein frame, is given by

$$\begin{aligned} ds_E^2 &= e^{2A(r)}(-dt^2 + dx_i dx^i) + e^{2B(r)}dr^2 + r^2 e^{2B(r)} d\Omega^2_{(8-p)} \\ &\equiv \bar{G}_{AB} dx^A dx^B \,, \end{aligned} \tag{2.1}$$

where $i = 1, \ldots, p$ and capital latin indices run over the 10 spacetime indices ($A, B = 0, \ldots, 9$). Here we focus on D3-branes: for $p = 3$ the functions appearing in (2.1) are given by [33]

$$A(r) = -\frac{1}{4}\ln Y \,, \qquad B(r) = \frac{1}{4}\ln[f_+ f_-] - A(r) \,, \tag{2.2}$$

where

$$\begin{aligned} Y &= \cosh(k\,h(r)) - c_2 \sinh(k\,h(r)) \,, \\ f_\pm &= 1 \pm \left(\frac{r_0}{r}\right)^4 , \\ h &= \ln\left[\frac{f_-}{f_+}\right] , \\ k &= \sqrt{\frac{5}{2} - c_1^2} \,. \end{aligned}$$

The corresponding dilaton $\phi(r)$ and 4-form $C^{(4)} = e^{\Lambda(r)} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$ are given by

$$\begin{aligned} \phi &= c_1 h(r) \,, \\ e^\Lambda &= -\tilde{\eta}(c_2^2 - 1)^{1/2} \frac{\sinh(k\,h(r))}{Y} \,, \end{aligned} \tag{2.3}$$

where $\tilde{\eta}$ is background non-BPS "brane" or "antibrane" charge.

Notice from (2.3) that $|c_2| \geq 1$, and also that $c_1^2 \leq 5/2$ since k must be real (see below). In the following we take $r_0 > 0$, and the resulting naked singularity at $r = r_0$ reflects the lack of supersymmetry of the system. (Hence $r \geq r_0$; for a discussion of the case in which $r_0 < 0$, see [33].) Indeed, recall that stable supergravity configurations are specified by two parameters corresponding to the ADM mass, $M_{\rm ADM}$, and the charge, Q, as required by Birkhoff's theorem. Here, though, a third parameter c_1 appears due to the non-trivial tachyon v.e.v. T. As a result Birkhoff's theorem must break down and this leads to the naked singularity. More specifically, for metric (2.1)

$$\begin{aligned} M_{\rm ADM} &= 2c_2 k N_3 r_0^4 \,, \\ Q &= 2\eta N_3 r_0^4 \sqrt{c_2^2 - 1} \,, \end{aligned} \tag{2.4}$$

where $N_3 = 5\omega_5 V_3 / 4\kappa_{10}^2$, $\omega_5 = \pi^3$, and for convenience we have wrapped the spatial world-volume directions on a torus of volume V_3 [33]. Notice from(2.4) that the parameter c_2 determines the charge of the background, i.e. when $c_2^2 = 1$ the background is neutral. In sections 3 and 4 we will mainly focus on that case. Also notice that the BPS relation $M_{\rm ADM} = Q$ is generally violated.

The physical interpretation of c_1 is that it is related to T [33]. Indeed one interpolates between a non-BPS brane when $c_1 = 0$, and a BPS-brane when $c_1 = \sqrt{5/2}$ ($k = 0$). The decay of the non-BPS brane can be viewed here as the trajectory in the parameter space when $c_1 \in [0, \sqrt{5/2}]$. In the following the value of c_1 is not specified, and we study the dynamics of a probe in the non-BPS background for any c_1 (thus corresponding to any stage of the decay).

We begin with the non-compactified case.

3. Brane dynamics and mirage cosmology

In this section we determine the dynamics of a probe BPS D3-brane in the non-compact background spacetime (2.1), neglecting any back-reaction effects of the probe onto the background spacetime.

We consider an infinitely straight brane, lying parallel to the x^μ ($\mu = 0, 1, 2, 3$) axes, but free to move along the r and φ^q ($q = 5, \ldots, 9$) axes. Its position, $X^A(t)$, is therefore given by

$$X^\mu = x^\mu \,, \qquad X^r(t) = R(t) \,, \qquad X^q(t) = \varphi^q(t) \,. \tag{3.1}$$

Below the Born-Infeld action is used to determine $R(t)$, $\varphi^I(t)$, and we will see that unless the BPS probe brane has sufficient angular momentum, it is attracted to the non-BPS brane and so the naked singularity at $r = r_0$ is not shielded.

For an observer living on the brane, the brane motion (3.1) leads to a FRW universe with scale factor $S(\tau)$ where τ is brane time. This effective "mirage" cosmology is due only to brane motion and not brane self-gravity [36]. In the resulting Friedmann equation, which we determine, the brane motion leads to "dark fluid" terms whose equation of state, ω, is calculated [35]. We also show that $0 \leq S(\tau) \leq 1$ when $c_2 \geq 1$ so that inflation may not occur on the brane in this case.

A crucial component which is missing in this mirage cosmology approach is brane self-gravity. This is included in section 4 where we compactify the background spacetime, and in that case we will see that inflation may occur. The scale factor $S(\tau)$ mentioned above and discussed in this section will still play a rôle when self-gravity is present.

3.1 Action and dynamics

We determine the probe BPS brane dynamics by solving the equations of motion coming from the action

$$S = S_{\rm DBI} + S_{\rm WZ} = -T \int d^4x e^{-\phi} \sqrt{-\det(\gamma_{\mu\nu} + (2\pi\alpha')F_{\mu\nu} - \hat{B}_{\mu\nu})} - qT \int \hat{C}_4 \,, \tag{3.2}$$

where T is the brane tension, α' the string tension, and $q = (-)1$ for a BPS (anti-)brane. In the DBI term, $\gamma_{\mu\nu}$ is the induced brane metric

$$\gamma_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \bar{G}^s_{AB}(X) \,, \tag{3.3}$$

where the superscript s refers to the string frame. For the remainder of this section we ignore the pull-back of the Neveu-Schwarz anti-symmetric two-form $\hat{B}_{\mu\nu}$ as well as the worldvolume anti-symmetric gauge fields $F_{\mu\nu}$. These will be considered in section 4 when we discuss bi-metric theories. The WZ term is

$$\int \hat{C}_4 = \frac{1}{4!} \int d^4x \epsilon^{\mu\nu\rho\sigma} \partial_\mu X^A \partial_\nu X^B \partial_\rho X^C \partial_\sigma X^D C_{ABCD} \tag{3.4}$$

with $\epsilon^{0123} = 1$.

In the static gauge (3.1), the induced metric is

$$\gamma_{00} = e^{\phi/2} \left(\bar{G}_{00} + \bar{G}_{rr} \dot{R}^2 + \bar{G}_s \dot{\varphi}^2 \right), \qquad \gamma_{ij} = e^{\phi/2} \bar{G}_{ij} , \tag{3.5}$$

where $\cdot = d/dt$ and $\dot{\varphi}^2 = h_{pq} \dot{\varphi}^p \dot{\varphi}^q$ with h_{pq} the metric on the 5-sphere. Since the functions appearing in action (3.2) depend on R, φ^q, and hence only on t (by virtue of (3.1)), one can therefore define a lagrangian $\mathcal{L}$ through

$$S = TV_3 \int dt \mathcal{L} , \tag{3.6}$$

where $V_3 = \int d^3x$ is the (infinite) spatial volume of the probe, and

$$\mathcal{L} = -\sqrt{a + b\dot{R}^2 + c\dot{\varphi}^2} + e , \tag{3.7}$$

where

$$a = e^{8A} , \qquad b = -e^{2(3A+B)} = \frac{c}{R^2} , \qquad e = -qe^{\Lambda} . \tag{3.8}$$

This lagrangian defines a conserved positive energy E and angular momentum l of the brane around the S^5 which are given by [36]

$$\begin{aligned} E &= \frac{\partial \mathcal{L}}{\partial \dot{R}} \dot{R} + \frac{\partial \mathcal{L}}{\partial \dot{\varphi}^p} \dot{\varphi}^p - \mathcal{L} , \\ l^2 &= h_{pq} \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_p} \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_q} . \end{aligned}$$

Hence

$$\begin{aligned} \dot{\varphi}^2 &= \frac{a^2 l^2}{c^2 (E+e)^2} , \\ \dot{R}^2 &= -\frac{a}{b} \left[1 + \frac{a}{c} \frac{(l^2 - c)}{(E+e)^2} \right] . \end{aligned} \tag{3.9}$$

Since a, b, c, and e are known functions of R, these equations can now be solved to determine the brane dynamics as seen by an observer in the bulk with time coordinate t. For an observer living on the brane, however, the line element is

$$\begin{aligned} ds_3^2 &= \gamma_{\mu\nu} dx^\mu dx^\nu \\ &= \gamma_{00}(t) dt^2 + \gamma_{ij} dx^i dx^j \\ &\equiv -d\tau^2 + S^2(\tau) d\mathbf{x}^2 , \end{aligned} \tag{3.10}$$

where the brane time τ and the scale factor $S(\tau)$ are given by

$$d\tau^2 = -\gamma_{00}(t) dt^2 , \qquad S^2 = e^{\phi/2} e^{2A} . \tag{3.11}$$

JHEP05(2002)016

Thus the observer on the brane appears to be living in a FRW universe with scale factor $S(R(\tau))$ arising from the brane motion. Notice that there is no term in action (3.2) of the form $\int d^4x\,\sqrt{-\gamma}\mathcal{R}_\gamma$ (where $\mathcal{R}_\gamma$ is the four-dimensional Ricci scalar). Thus the above scale factor is only due to brane motion and not due to self-gravity on the brane: in section 4 we show how self-gravity can be included. In that case the scale factor $S(\tau)$ will still be important.

From eqs. (3.11) and (3.9) we find

$$d\tau^2 = e^{-6A+\frac{\phi}{2}}\frac{a^2}{(E+e)^2}dt^2 \tag{3.12}$$

so that $R^{'2}$ (where $' = d/d\tau$) is given by

$$R^{'2} = -\frac{1}{abc}e^{6A-\frac{\phi}{2}}\left[c(E+e)^2 + a(l^2-c)\right]. \tag{3.13}$$

Finally the Friedman equation on the brane is

$$H^2 = \left(\frac{1}{S}\frac{dS}{d\tau}\right)^2 = R^{'2}\left[\frac{dA}{dR} + \frac{1}{4}\frac{d\phi}{dR}\right]^2 \equiv \frac{8\pi}{3}\rho_{\rm eff} \tag{3.14}$$

which defines an effective energy density. The corresponding equation of state, which we determine in section 3.3, is

$$\omega = \frac{p_{\rm eff}}{\rho_{\rm eff}} = -\left(1 + \frac{1}{3}\frac{S}{H^2}\frac{\partial H^2}{\partial S}\right). \tag{3.15}$$

Clearly in order to find ω, we need first to determine the brane dynamics, $R(t)$. This will depend on the brane energy E and angular momentum l as well as the parameters c_1, c_2 characterising the non-BPS bulk.

3.2 Brane dynamics

It is helpful to use the analogy with particle dynamics and to study the brane motion via the two effective potentials

$$V^t(R,l,E) = E - \frac{1}{2}\dot{R}^2\,, \qquad V^\tau(R,l,E) = E - \frac{1}{2}R'^2\,, \tag{3.16}$$

where $\dot{R}$ and R' are given respectively in equations (3.9) and (3.13). The first potential, V^t, describes the brane motion as seen by a bulk observer. The second, V^τ, determines the brane dynamics as seen by an observer on the brane. Allowed regions of R are ones for which $V^{t,\tau}(R,l,E) \leq E$. Notice the dependence of the potentials on E and l.

When the probe is very far from the non-BPS brane, $R \to \infty$, it is straightforward to show from (3.9) and (3.13) that for all c_1, c_2 and l,

$$\dot{R}^2 \longrightarrow 1 - \frac{1}{E^2}\,, \qquad \text{and} \qquad R^{'2} \longrightarrow E^2 - 1\,. \tag{3.17}$$

Hence both in background time t and brane-time τ, the probe will be unable to escape to infinity if $E < 1$. In other words, for $E < 1$ the BPS probe brane is always bound to the non-BPS brane and, depending on l (see below) will eventually be absorbed by it. On the other hand, if $E > 1$ the probe will be able to escape. The limiting case is when $E = 1$, for which the kinetic energy of the probe vanishes at infinity where there is no force on the probe. In the BPS limit (of the non-BPS background, as discussed in [33]), both $\dot{R}$ and R' vanish for all R when $E = 1$, so that R is a flat direction, as expected.

To determine the brane dynamics for $r_0 \leq R < \infty$ consider first V^τ which is the relevant potential for an observer living on the brane. Very similar comments to the ones below also hold for V^t. We focus on the neutral limit, $c_2 = 1$, which will be of most interest when self-gravity is included in section 4. (Notice that when $c_2 = \pm 1$ then $e = 0$ and the coupling between the BPS probe and the bulk RR field vanishes. Below we see that for $c_2 = -1$ the scale factor $S(\tau)$ is badly defined.) Using (3.16) and (3.13) as well as the definitions of a, b and c in (3.8) gives

$$V^\tau(R, l, E) = E - \frac{1}{2}\frac{(f_+)^{(3k+c_1-1)/2}}{(f_-)^{(3k+c_1+1)/2}}\left[E^2 - \left(\frac{f_-}{f_+}\right)^{2k} - \frac{l^2}{r^2}\frac{(f_-)^{k-1/2}}{(f_+)^{k+1/2}}\right]. \tag{3.18}$$

When the probe has zero angular momentum, $l = 0$, and when $k > 0$, eq. (3.18) gives $V^\tau(r_0, 0, E) \to -\infty$. Thus for all E there is an infinitely deep potential well at $R = r_0$: the probe is attracted to the singularity which is therefore not protected. If $E < 1$ then the probe is bound in the region

$$r_0 \leq R \leq \bar{r} = r_0 \left(\frac{1 + E^{1/k}}{1 - E^{1/k}}\right)^{1/4}.$$

When $l \neq 0$, a centrifugal potential can develop and this may shield the singularity. From (3.18) notice that if $k < 1/2$ then $V^\tau(r_0, l, E) \to +\infty$ for all E so that the singularity is shielded. If $k \geq 1/2$, however, $V^\tau(r_0, l, E) \to -\infty$ so that the probe may reach the singularity. Despite that some regions of R may be excluded even for $k \geq 1/2$, i.e. if l is sufficiently large $l > l_c$ where l_c is a critical angular momentum which depends on E, r_0 and c_1, then one can show that there are indeed such excluded regions [35]. These results are shown in figure 1.

3.3 Mirage cosmology

How is this dynamics reflected in the Friedmann equation on the brane? From eq. (3.11), the brane scale factor $S(\tau)$ is given by

$$S(\tau)^4 = e^\phi e^{4A} = \left(\frac{f_-}{f_+}\right)^{c_1}\frac{1}{Y}, \tag{3.19}$$

where Y is a function of c_2. If $c_2 < -1$, there are regions of R for which $S^4 < 0$ thus leading to an imaginary scale factor: these regimes cannot be physical and so we take $c_2 \geq 1$. In that case the scale factor is bounded by

$$0 \leq S \leq 1$$

since $S = 0$ as $R \to r_0$ and $S = 1$ as $R \to \infty$. Thus there can be no inflation on the brane in this mirage cosmology approach.

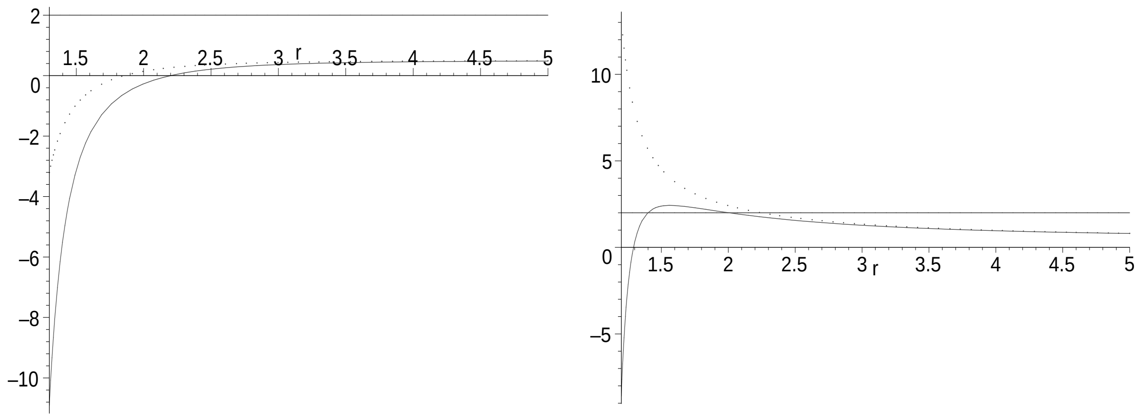

Figure 1: Graphs of $V^\eta(r,l,E)$ as a function of r. For simplicity we have chosen the parameters to be $q = 1 = \tilde{\eta}$ and $r_0 = c_2 = 1$. The probe energy is $E = 2$ corresponding to the horizontal line. The LH figure is for a probe with zero angular momentum $l = 0$ — the dotted curve has $k = 0.44 < 1/2$, and the other $k = 3/2$. In the RH figure, $l = 4 > l_c$.

Again we consider the neutral limit $c_2 = +1$. Then

$$S^4 = x^{c_1+k}\,, \qquad \text{where } x = \frac{f_-}{f^+}\,. \tag{3.20}$$

Furthermore, by definition of H in (3.14) one has

$$H^2 = \frac{1}{2}\frac{(c_1+k)^2}{r_0^2}\frac{(1-x^2)^{5/2}}{x^{5+3k+c_1/2}}\left[E^2 - x^{2k} - \frac{l^2}{r_0^2}\frac{(1-x^2)^{1/2}}{2x^{1/2}}\right] \tag{3.21}$$

which, combined with (3.20) gives $H^2(S)$. Hence ω defined in (3.15) can be calculated. There are two difficulties though: firstly $H^2(S)$ is not a sum of terms of the form S^{-n} for some integer n, since functions such as $(1-x^2)^{5/2} = (1 - S^{8/(k+c_1)})^{5/2}$ appear in H^2. Secondly, c_1 can take any continuous value between 0 and $\sqrt{5/2}$ and so ω can also take any value.

To overcome the first problem consider the limit $S \ll 1$ (and hence $R = r_0 + \epsilon$). A Taylor expansion of H^2 then contains terms of the form

$$x^{(\alpha k + \beta c_1 + \gamma)/2} = S^{2(\alpha k + \beta c_1 + \gamma)/c_1 + k} \tag{3.22}$$

for integer α, β and γ. Surprisingly two values of c_1 are picked out for which this expansion of H^2 contains terms in S to an integer power, they are[1]

$$c_1 - k = \pm 1 \stackrel{1}{\Longrightarrow} c_1 = \frac{3}{2}, \frac{1}{2}\,. \tag{3.23}$$

When $c_1 = 3/2$, the Friedmann equation contains "dark fluids" with equation of state $\omega = 2, 2/3, -2/3$. These are sourced by the angular momentum l of the brane. There are

[1]These two values do not seem to correspond to any special case of the non-BPS background metric (2.1).

also terms with $\omega = 5/3, -1$ which are sourced by the energy E of the brane, and terms in $\omega = 1,\ 1/3$ which are independent of E and l. Note that there is a term behaving as radiation, as well as a cosmological constant term, but there is none behaving as matter.

For $c_1 = 1/2$ the main difference is that there is now a dark "matter" term $\propto S^{-3}$ and this is sourced by the angular momentum l. As far as we are aware, this is the first background for which such a dark "matter" term appears in mirage cosmology.

4. Non-BPS Brane Inflation

In the previous section we described the brane dynamics in non-compact space-time. Now we concentrate on the low energy dynamics of the same system, i.e. a probe brane in the vicinity of a non-BPS brane, when the six extra dimensions have been compactified. We do not assume any particular form for the compactification manifold — on the contrary we only need to consider a patch where the non-BPS brane and the probe are embedded. In that patch the geometry is curved by the non-BPS brane while the effective field theory on the probe brane is subject to two competing effects. Firstly gravity now propagates on the brane, hence modifying the induced metric from the previously studied mirage cosmology case. Secondly the motion of the probe in the background of the non-BPS brane modifies the geometry of the probe. The combined effect can be understood by analysing the equations of motion of the low energy effective field theory below the compactification scale.

4.1 Gravitons on the Brane

Consider now the 10d metric (in the Einstein frame)

$$ds_E^2 = e^{2A} g_{\mu\nu} dx^\mu dx^\nu + e^{2B}(dr^2 + r^2 d\Omega_5^2) \equiv G_{AB} dx^A dx^B \,, \tag{4.1}$$

where $g_{\mu\nu}$ is any 4d metric, as opposed to the Minkowski metric considered in (2.1). For simplicity we now focus on the neutral case, $|c_2| = 1$, in which case $F^2 = 0$ from eq. (2.3). We will determine the equations of motion for $g_{\mu\nu}$ and show that one can construct a whole family of solutions. The previously described solution $g_{\mu\nu} = \eta_{\mu\nu}$ is only the simplest case. As long as we do not compactify six dimensions, the dynamics of $g_{\mu\nu}$ are frozen due to the infinite effective four-dimensional Planck mass, i.e. four-dimensional gravity decouples. In the previous section we have considered the mirage dynamics where the fluctuations of the four-dimensional metric $g_{\mu\nu}$ have been frozen. As soon as we compactify, the effective four-dimensional Planck mass becomes finite allowing one to study the gravitational dynamics of $g_{\mu\nu}$.

Using the relation

$${}^{10}\mathcal{R}(G) = {}^{4}\mathcal{R}(g) e^{-2A} + {}^{10}\mathcal{R}(\bar{G}) \,, \tag{4.2}$$

it follows that the 10d SUGRA action for the metric (4.1) reduces to

$$S_{\rm sugra} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-\bar{G}} \sqrt{-g}\, e^{-2A}\ {}^{4}\mathcal{R}(g) \tag{4.3}$$

(see also section 4.2. From this action, the equations of motion for $g_{\mu\nu}$ reduce to the 4d Einstein equations

$$ {}^4\mathcal{R}_{\mu\nu} - \frac{{}^4\mathcal{R}}{2} g_{\mu\nu} = 0 \,. \tag{4.4} $$

Hence the metric $g_{\mu\nu}$ can be any four-dimensional solution of the vacuum Einstein equations — a Schwarzchild metric, for instance, leads to a solution of the 10d equations of motion.

Now consider small perturbations of the flat metric $\eta_{\mu\nu}$, i.e. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. This describes the moduli space of solutions where $h_{\mu\nu}$ satisfies $\eta^{\mu\nu} h_{\mu\nu} = 0$, $\partial_\mu h^\mu_\nu = 0$ and is massless

$$ {}^4\Box h_{\mu\nu} = 0 \,. \tag{4.5} $$

Hence massless gravitons $h_{\mu\nu}$ can propagate on the brane. Nevertheless, in this uncompactified case the effective four-dimensional Planck mass, defined through

$$ \frac{1}{2\kappa_4^2} = \frac{1}{2\kappa_{10}^2} \int \sqrt{-\bar{G}} e^{-2A} dr d\Omega_5 \tag{4.6} $$

diverges, implying the decoupling of self-gravity on the brane. In the remainder of this section we therefore consider the compactified case in which the compactification is over a large 6d manifold $\mathcal{M}$, and gravity does not decouple anymore.

4.2 Effective 4D action with self-gravity

Here we compactify the theory and study the effective field theory at low energy below the compactification scale — as explained above, this should contain self-gravity. The low energy action therefore comprises two terms: (first there is the action obtained by dimensional reduction of the warped background created by the non-BPS brane. Second we include the Born-Infeld action (3.2) reflecting the low energy dynamics of the transverse modes to the probe. In order to maintain the neutrality of the background we will focus on the case $|c_2| = 1$.

Before doing so, let us discuss the compactification in general. Our starting point is the metric (4.1) where $g_{\mu\nu} dx^\mu dx^\nu = \eta_{ab} d\xi^a d\xi^b$ in a local frame $d\xi^a$. We make the ansatz that both the form F as well as the dilaton ϕ coincide with the solution obtained previously in the local frame specified by $d\xi^a$. This amounts to specifying that locally the solution of the previous section is valid. Hence

$$ F_{r\mu_0\cdots\mu_3} dr \wedge dx^{\mu_0} \wedge \cdots \wedge dx^{\mu_3} = \Lambda' e^{\Lambda} dr \wedge d\xi^{a_0} \wedge \cdots \wedge d\xi^{a_3} \,. \tag{4.7} $$

It is through $g_{\mu\nu}$ that we will be able to describe self-gravity on the brane. The dynamics at low energy are determined by

$$ \begin{aligned} S = &\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left({}^{10}\mathcal{R}(G) - \frac{1}{2} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2\dot{5}!} F^2 \right) - \\ &- T \int d^4x e^{-\phi} \sqrt{-\gamma} - Tq \int d^4x C_4 \,, \end{aligned} \tag{4.8} $$

where the first term is the bulk 10d supergravity action, and the second is the DBI action (3.2). We will integrate out the above action over the 6 extra dimensions which are compactified on $\mathcal{M}$, and hence obtain an effective 4D action which includes brane self-gravity through the appearance of the 4D Ricci scalar ${}^4\mathcal{R}(g)$. We assume that the manifold $\mathcal{M}$ is fixed, i.e. the moduli have been stabilized by an unknown mechanism. On using (4.6), the bulk part of action (4.8) becomes

$$S_{\rm bulk} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa_4^2}\,{}^4\mathcal{R}(g)\right] - \frac{1}{4.5!\kappa_{10}^2}\int d^4x\sqrt{-g}\left(\int_{\mathcal{M}} dr d\Omega_5 \sqrt{-\bar{G}}F^2\right), \qquad (4.9)$$

where the 4D planck mass is defined analogously to (4.6), namely

$$\frac{1}{2\kappa_4^2} = \frac{1}{2\kappa_{10}^2}V_6\,, \qquad (4.10)$$

where V_6 is the now finite volume of the 6d compact space;

$$V_6 \equiv R_c^6 = \int_{\mathcal{M}} \sqrt{-\bar{G}}e^{-2A} dr d\Omega_5\,. \qquad (4.11)$$

Notice that κ_4^2 is a function of c_1, and also that

$$F^2 = -5!\left(\Lambda' e^{\Lambda}\right)^2 e^{-8A-2B} = -5!\frac{64\xi^2k^2(c_2^2-1)r_0^8}{Y^{5/2}r^{10}f_+^{5/2}f_-^{5/2}}\,. \qquad (4.12)$$

Notice that the factors of A and B appear due to the non-trivial contraction with the background metric. This defines the gravitational part of the effective action. Notice that there is a negative cosmological constant related to the background field strength. Hence in the neutral case ($c_2 = \pm 1$), both F^2 and the coupling of the probe to the four form vanish leading to the absence of the cosmological constant. On the other hand, in the charged case the resulting action contains a negative cosmological constant leading to AdS$_4$ in the absence of the probe.

Now consider the brane part of the action. We work in the isotropic gauge where

$$X^\mu = x^\mu \qquad (4.13)$$

but as opposed to section 2 the brane is now allowed to be bent. Hence

$$X^r = R(x^\mu)\,, \qquad X^p = \varphi^p(x^\mu)\,, \qquad (4.14)$$

and the induced metric is thus

$$\gamma_{\mu\nu} = G^s_{\mu\nu} + G^s_{rr}\frac{\partial R}{\partial x^\mu}\frac{\partial R}{\partial x^\mu} + G^s_{pq}\frac{\partial\varphi^p}{\partial x^\mu}\frac{\partial\varphi^q}{\partial x^\nu}\,. \qquad (4.15)$$

We will focus on the motion in the r direction (and so set $l = 0$ in the notation of the previous section). In the DBI action, we need to consider the expansion of $\sqrt{-\gamma}$: if we assume that the brane moves slowly so that the square root part of the DBI action can be expanded one gets, to leading order,

$$\sqrt{-\gamma} = e^{4A}e^{\phi}\sqrt{-g}\left[1 + \frac{1}{2}e^{2B-2A}g^{\mu\nu}\frac{\partial R}{\partial x^\mu}\frac{\partial R}{\partial x^\nu}\right]. \qquad (4.16)$$

(Below we will see that R is very closely related to the inflaton on the brane, and that this assumption of a slowly moving brane is consistent with the slow-roll conditions.) Thus

$$S_{\rm brane} = -T \int d^4x \sqrt{-g} e^{4A} \left[1 + \frac{1}{2} e^{2B-2A} g^{\mu\nu} \partial_\mu R \partial_\nu R\right] - qT \int d^4x \sqrt{-g} e^{\Lambda} . \tag{4.17}$$

This is a non-linear sigma model for R with a non-vanishing potential

$$V(R) = T\left(e^{4A} - qe^{\Lambda}\right) \tag{4.18}$$

due to the absence of supersymmetry.[2] This potential is valid generically for any motion of a D3 probe in a non-trivial gravitational background. In the supersymmetric case the contributions from the Wess-Zumino coupling and the DBI cancel. Here the absence of cancellation leads to a non-trivial dynamics for the interbrane distance. Notice also the wave-function normalization term

$$Z(R) = Te^{2B+2A} . \tag{4.19}$$

It is convenient to use fields with mass dimension one by putting

$$\phi = \frac{R}{l_s^2} , \tag{4.20}$$

where l_s is the string length scale. We now focus on the neutral case so that it follows from (4.9) and (4.17) that the effective 4D action is given by

$$S^4_{\rm eff} = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa_4^2}\, {}^4\mathcal{R}(g) - \frac{1}{2} Z(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) . \tag{4.21}$$

Below we study this action and we will be interested in the supergravity regime where the two branes are far apart $R \gg r_0$ leading to

$$\begin{aligned} V(\phi) &= T\left(1 - \frac{2kr_0^4}{l_s^8 \phi^4}\right) , \\ Z(\phi) &= l_s^4 T , \end{aligned}$$

where we have neglected higher order terms than $1/\phi^4$ in Z.

In the next section we will analyse cosmological solutions to the equations of motion coming from action (4.21). Before that, notice an intriguing point — the induced metric on the brane $\gamma_{\mu\nu}$ differs from the metric $g_{\mu\nu}$ which is responsible for Einstein gravity at low energy. Furthermore matter couples to the induced metric as can be seen by including a matter term in the DBI corresponding to the pull-back of the U(1) field strength

$$F_{\mu\nu} = F_{AB} \frac{\partial X^A}{\partial x^\mu} \frac{\partial X^B}{\partial x^\nu} . \tag{4.22}$$

[2] Note that this potential is not directly related to the effective potentials $V^{\tau,t}$ discussed in the previous section.

Action (3.2) leads to the kinetic terms

$$(2\pi\alpha')^2\frac{T}{8}\int d^4x\sqrt{-\gamma}F^{\mu\nu}F^{\rho\delta}\gamma_{\mu\rho}\gamma_{\nu\delta} \tag{4.23}$$

so that matter does indeed couple to the induced metric in the string frame. Thus we have a bi-metric theory [39, 40] where

$$\gamma_{\mu\nu} = e^{\phi/2}e^{2A}g_{\mu\nu} + e^{\phi/2}e^{2B}\partial_\mu R\partial_\nu R\,. \tag{4.24}$$

As soon as the brane stops moving, the theory stops being a bi-metric theory.

4.3 Inflationary solution and induced metric

We search for inflationary solutions of (4.21) of the form

$$g_{\mu\nu} = \mathrm{diag}(-1, a^2(t), a^2(t), a^2(t))\,. \tag{4.25}$$

It should be noted, however, that a is not the scale factor as seen by an observer living on the brane. The metric on the brane is just the induced metric $\gamma_{\mu\nu}$ as discussed above and as given in (4.24). Assuming now, for simplicity, that $R = R(t)$ only, the metric on the brane is therefore given by

$$\begin{aligned} ds^2 &= dt^2(e^{\phi/2}e^{2A}g_{00} + e^{\phi/2}e^{2B}\dot{R}^2) + e^{\phi/2}e^{2A}g_{ij}dx^i dx^j \\ &\equiv -d\tau^2 + \tilde{a}^2(\tau)dx^2\,, \end{aligned}$$

where $\cdot = d/dt$ and

$$\tilde{a}(\tau) \equiv S(\tau)a(\tau)\,. \tag{4.26}$$

It combines the mirage cosmology brane factor $S^2(\tau) = e^{\phi/2}e^{2A}$ which is due to the motion of the brane, and a which arises from the self-gravity on the brane. As usual, inflation will lead to an exponential expansion on the brane $a \sim e^{Ht}$.

Since matter on the brane couples to $\gamma_{\mu\nu}$, whilst gravity couples to g_{ab}, predictions regarding the CMB and the spectrum of perturbations should in principle be made by calculating with respect to the scale factor $\tilde{a}$ and brane-time τ, since particle and photon geodesics are governed by the metric $\gamma_{\mu\nu}$. Thus one should replace g_{ab} in the effective action (4.21) by $\gamma_{\mu\nu}$, i.e. inverting (4.24). This, however, would lead to a rather complicated scalar-tensor action, and in that frame the equations of motion are not transparent. Moreover the generation of quantum fluctuations is easier to understand in the Einstein frame where all the usual formalism applies. We will comment on the coupling to matter later and from now on work in the Einstein frame with action (4.21) and time t (as opposed to brane time τ). Eventually, in the inflationary phase, the difference between t and τ can be seen to be negligible due to the slow roll condition. Similar actions were taken as the starting point of references [18, 19, 20, 24, 27, 29].

The equations of motion coming from the effective action are

$$\begin{aligned} &\ddot{\phi} + 3H\dot{\phi} + \frac{1}{2}\left(\frac{Z'}{Z}\right)\dot{\phi}^2 + \frac{V'}{Z} = 0\,, \\ &\qquad\qquad H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa_4^2}{3}\left(\frac{1}{2}Z\dot{\phi}^2 + V\right). \end{aligned}$$

Assuming that

$$\frac{1}{2}Z\dot{\phi}^2 \ll V\,, \qquad \ddot{\phi} \ll 3H\dot{\phi} \tag{4.27}$$

i.e. in the slow roll regime, yields

$$H^2 = \frac{\kappa_4^2}{3}V \simeq \frac{\kappa_4^2 T}{3} \tag{4.28}$$

leading to

$$a(t) = a(t_0)e^{\sqrt{\frac{\kappa_4^2 T}{3}}(t-t_0)}\,. \tag{4.29}$$

As

$$\dot{\phi} = -\frac{V'}{3HZ}\,, \tag{4.30}$$

where V' is positive, it follows that during inflation ϕ (or equivalently R) decreases. Notice that the second scale factor S actually decreases as R decreases. Since the scale factor on the brane is actually Sa, the overall effect will still be one of inflation since S is only a powerlaw in t. From (4.30) we find that

$$\phi^6 = \phi_0^6 - Q(t-t_0)\,, \tag{4.31}$$

where

$$Q = \frac{48kr_0^4}{\sqrt{3}l_s^{12}\kappa_4\sqrt{T}}\,. \tag{4.32}$$

The slow-roll regime is valid provided that

$$\eta = M_{pl,4}^2\frac{V''}{V} \ll 1\,, \qquad \epsilon = \frac{1}{2}M_{pl,4}^2\left(\frac{V'}{V}\right)^2 \ll 1\,. \tag{4.33}$$

Here ϵ is always smaller than η so we only consider the first slow roll condition. Imposing the first condition gives

$$|\eta| = M_{pl,4}^2\frac{40kr_0^4}{l_s^8\phi^6} \ll 1 \tag{4.34}$$

or equivalently

$$k \ll \frac{1}{40}\frac{r_{\rm ini}^6}{M_{pl,4}^2 l_s^4 r_0^4}\,, \tag{4.35}$$

where we have taken the initial brane position $R = r_{\rm ini} \gg r_0$ consistently with our supergravity approximation. Using now

$$\begin{aligned} T &= \frac{1}{g_s(2\pi)^3 l_s^4}\,, \\ \kappa_{10}^2 &= \frac{1}{2}(2\pi)^7 l_s^8 \end{aligned} \tag{4.36}$$

gives

$$k \ll \frac{(2\pi)^8}{20}\frac{l_s^4}{r_0^4}\left(\frac{r_{\rm ini}}{R_c}\right)^6\,. \tag{4.37}$$

Thus k is undetermined as long as r_0 remains a free parameter of the supergravity approximation.

In order to specify the range of validity of inflation in the non-BPS scenario, we need to compare the time taken by the non-BPS to decay with the typical inflationary time scale. Inflation typically takes place within

$$t_{infl} = \frac{1}{H} \tag{4.38}$$

whereas the decay time for the non-BPS brane is assumed to be infinite in the supergravity approximation. Indeed, in the approach we use here, the tachyon parameter c_1 is a constant parameter and not a time-dependent modulus [33]. A more appropriate estimate of the decay time in the inflationary regime can be obtained from

$$t_{\rm decay} = \frac{1}{M_{\rm ADM}}, \tag{4.39}$$

where $M_{\rm ADM}$ is the ADM mass of the non-BPS brane [33]. When the brane is flat the ADM mass is infinite reflecting the fact that it has a finite mass per unit volume

$$\frac{M_{\rm ADM}}{V_3} = \frac{5}{2^7\pi^4} k_{\rm max} \frac{r_0^4}{l_s^8}, \tag{4.40}$$

where $k_{\rm max} = \sqrt{5/2}$. Now when the brane inflates, the volume of space-time is of order of $V_3 = H^{-3}$ implying that

$$t_{\rm decay} = \frac{2^7\pi^4}{5k_{\rm max}} \frac{l_s^8}{r_0^4} H^3 . \tag{4.41}$$

Imposing now that $t_{decay} \gg t_{inf}$ leads to

$$r_0^4 \ll \frac{1}{40.6^2.\pi^4} \frac{1}{g_s^2} \frac{1}{M_{pl,4}^4} . \tag{4.42}$$

Notice that r_0 is linked to the 4d Planck mass. In the following we will take $r_0 = 1/M_{pl,4}$ which satisfies the above inequality at small string coupling. This leads to

$$\frac{k}{k_{\rm max}} \ll \frac{9}{2} (2\pi)^{12} g_s^2 l_s^4 M_{Pl,4}^4 \left(\frac{r_{\rm ini}}{R_c}\right)^6 . \tag{4.43}$$

We will discuss the range of values for k in the next section.

4.4 Reheating

In this subsection we describe the end of the inflationary era and its connection with the cosmological eras. First of all inflation on the brane stops when $\eta \sim 1$. We find that

$$Q(t_{\rm end} - t_0) = \frac{r_{\rm ini}^6}{l_s^{12}} - \frac{40 M_{pl,4}^2 r_0^4}{l_s^8} . \tag{4.44}$$

After the end of inflation the brane starts rolling fast towards the non-BPS brane. When the non-BPS brane decays the probe has to be far enough from the non-BPS brane in order to trust the supergravity approximation: this leads to

$$g_s^2 \gg \frac{2^{21/2}\pi^5}{15} \frac{k}{k_{\rm max}} \frac{l_s^{12}}{r_{\rm ini}^6 R^6} . \tag{4.45}$$

This condition is always satisfied for the range of parameters compatible with the COBE normalization. When the non-BPS brane decays, it leads to bulk radiation in the Minkowski vacuum. Moreover we know that $k_{\rm decay}=0$ implying that the potential vanishes altogether

$$V_{\rm decay}=V_0+T\equiv 0\,, \tag{4.46}$$

where V_0 is the contribution at low energy to the tachyon condensation process. It goes beyond the supergravity approximation and should encompass the opens string degrees of freedom leading to the decay of the non-BPS brane. The vanishing of V_0+T is similar to the Sen conjecture for non-BPS branes [9]. Here it can be viewed as the vanishing of the cosmological constant at low energy due to the overall supersymmetry of the configuration. The only trace of the decay may appear from the coupling of the bulk radiation to the brane YM fields leading to a radiation dominated era.

Once the non-BPS brane decays, the brane stops moving and starts expanding only due to the presence of matter on the brane. The background metric being flat Minkowski space, the ten-dimensional metric becomes

$$ds^2_{10}=dx_A dx^A-dt^2+a^2(t)dx^i dx_i\,, \tag{4.47}$$

where $A=1,\ldots,6$ and $i=1,\ldots,3$. This implies that the induced metric and the Einstein metric become equivalent. Moreover the six extra dimensions play no role in the four-dimensional dynamics. We can therefore apply the usual formalism to compute the effect of the initial quantum fluctuations on the CMB results. Indeed once inflation stops, the cosmology of the D3-brane is solely dictated by the presence of matter on a static brane. In that case we can follow the evolution of the brane through the usual cosmological eras and use the CMB results in order to calibrate the underlying physics of the brane inflation.

In particular we can use the COBE normalization [41]–[45] specifying th magnitude of the density perturbations on large scales in order to evaluate the compactification scale. This gives

$$M^{-3}_{pl,4}\frac{V^{3/2}}{V'}\sim M^{-3}_{pl,4}\frac{V_6}{kl_s^9}\sim 10^{-5}\,. \tag{4.48}$$

Now

$$l_s^9\sim\left(\frac{V_6}{M^2_{pl,4}}\right)^{9/8}, \tag{4.49}$$

and on writing $M_c^6=V_6^{-1}$, one has that

$$M_c\sim 10^{12}\,{\rm GeV}\,. \tag{4.50}$$

Using the relation between the four-dimensional Planck mass, the compactification volume and the string scale we deduce that

$$M_s\sim M^{1/4}_{pl,4}M_c^{3/4}\sim 10^{14}\,{\rm GeV}\,, \tag{4.51}$$

so that the compactification scale is two orders of magnitude smaller than the string scale. Notice that $r_0\ll R_c$ as it should in order to consider that the non-BPS brane distorts only locally and weakly the geometry of the compact manifold $\mathcal{M}$.

Let us now consider the observable consequences of the primordial inflationary era due to the presence of a non-BPS brane. Two crucial tests have to be overcome. First of all the number of e-folds must be large enough. The number of e-folds

$$N = \int_{t_{\rm ini}}^{t_{\rm end}} H dt \tag{4.52}$$

is determined by the initial position of the probe. Indeed

$$N = \frac{5}{6}\left(\frac{r_{\rm ini}}{r_{\rm end}}\right)^6 . \tag{4.53}$$

Using

$$r_{\rm end}^6 = 40 k r_0^4 l_s^4 M_{pl,4}^2 \tag{4.54}$$

this implies that

$$r_{\rm ini}^6 = 48 N k r_0^4 l_s^4 M_{pl,4}^2 \, . \tag{4.55}$$

Taking r_0 to be of the order of $M_{pl,4}^{-1}$, i.e. saturating its upper bound, leads to

$$\frac{r_{\rm ini}}{r_0} \sim \left(\frac{M_{pl,4}}{M_s}\right)^{2/3} . \tag{4.56}$$

Notice that the ratio of the Planck scale to the string scale is related to the COBE normalization, i.e. 10^{-5}, we find that

$$\frac{r_{\rm ini}}{r_0} \sim 10^3 \, . \tag{4.57}$$

Of course this justifies the supergravity approximation. Notice that all the different scales of the problem have been determined from two inputs. First of all the non-BPS brane scale r_0 is taken to be of the order the Planck scale, this to guarantee that inflation stops well before the decay of the non-BPS brane. Then the COBE normalization fixes the volume of the compactification manifold.

Now we can check that inflation takes place irrespective of the value k. Hence we find that inflation is generic in the non-BPS scenario.

Within the supergravity approximation the spectral index of fluctuations can be computed. It is given by [24]

$$n_s - 1 = -\frac{5}{3N} \, . \tag{4.58}$$

Of course this is not a prediction of the model as N is left undetermined, i.e. N depends on the initial position of the probe. The model does not explain why the initial conditions should be such that $N = 60$.

5. Summary and Conclusions

We have described the dynamics of a BPS probe in the background of a non-BPS brane. We have shown that inflation can only take place after compactifying on a six-dimensional manifold. The resulting inflation occurs for any non-BPS brane. It ends well before the final decay of the non-BPS brane; the only fine-tuning being the choice of the initial positions of the branes which determines the spectral index.

Of course the essential ingredient here is the lack of supersymmetry which leads to a non-trivial potential for the inter-brane distance. The actual decay of the non-BPS brane may only play a role in the reheating phase where radiation starts dominating due to the impinging radiation resulting from the non-BPS brane annihilation. It would be very interesting to carry out the same analysis for stable non-BPS brane as obtained in compact spaces directly. Of course one would have to motivate the reheating phase differently.

As it stands the brane technology seems to lead to an almost unique form of the inflationary potential, an inverse power law. This is a tremendous improvement over the four-dimensional approach where all the potentials seem to be ad hoc and difficult to motivate. On the other hand such a potential is reminiscent of the potentials used to describe quintessence. It would be very interesting if stable non-BPS configurations [46, 47, 48] could provide a unique description of both inflation and quintessence. This is left for future work.

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